Light orbiting a five-dimensional black hole

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Outline

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Photon spheres

Given a particular spacetime background, we can study the movement of particles (or light) moving in this background using the geodesic equations.

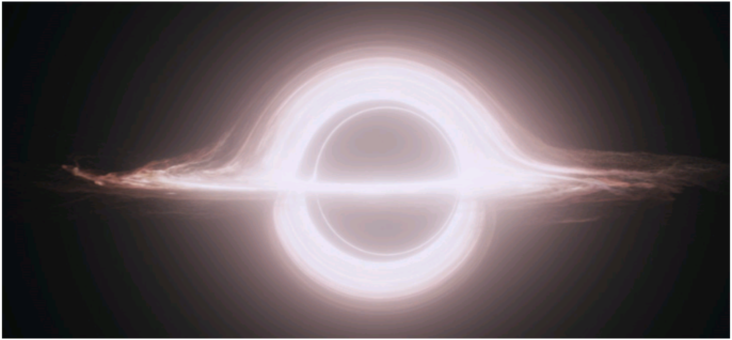

Figure: Created by Double Negative team using DNGR, and TM & ©Warner Bros. Entertainment Inc.

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In this talk, we will be investigating null geodesics around the simplest rotating black hole in 5-dimensions.

Why do we care about photon spheres?

- Optical appearance of stars undergoing gravitational collapse
- Shadow of black hole
- How the night sky would look to an observer near a black hole

Why do we care about higher-dimensional black holes?

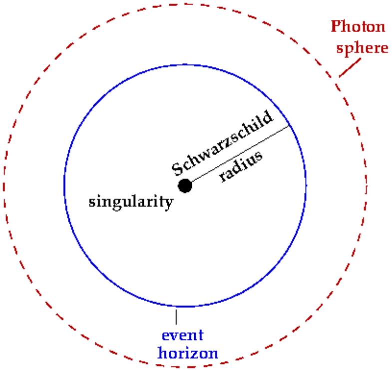
- Gauge/gravity correspondence relates 5 dimensional black holes to 4 dimensional Quantum Field Theory
- They arise naturally in string theory and brane-world scenarios
- They can be used to construct metrics on compact Sasaki-Einstein manifolds
- Their novel features help us understand gravity
- Their novel features help us understand string theory
- The first microscopic string theoretic calculation of black hole entropy was done in five dimensions
- Testing ground for mathematical relativity
- Seed solutions for supergravity dualities

Overview

The photon sphere in Schwarzschild

The Schwarzschild metric describes the spacetime around a spherically symmetric black hole. The metric has an event horizon, located at $r = 2M$.

Null geodesics with constant radius exist only at $r = 3M$. The collection of all orbits form the *photon sphere* of the black hole.

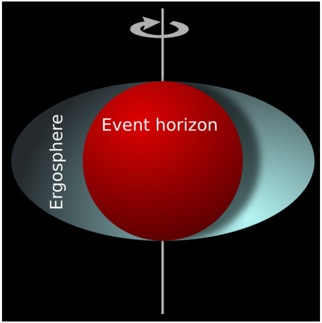


The Kerr spacetime

The Kerr metric describes the spacetime outside a rotating, axisymmetric black hole.

Frame dragging causes objects close to the black hole to corotate with the black hole, even if they were initially counterrotating.

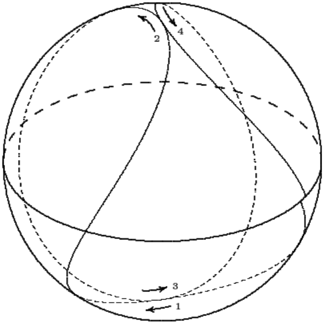
Spacetime itself is dragged around the black hole.



This frame dragging changes the properties of photon orbits and makes them much more interesting.

Orbits in Kerr: Non-planar!

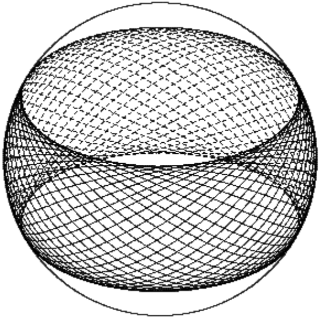
Photon spheres in Kerr can be non-planar!



Spherical Photon Orbits Around a Kerr Black Hole, E. Teo, Gen. Rel. and Grav. **35**

Orbits in Kerr: Maximum latitude!

They can form quasiperiodic patterns and have maximum latitudes.



Spherical Photon Orbits Around a Kerr Black Hole, E. Teo, Gen. Rel. and Grav. **35**

Features of photon orbits in Kerr

- Non-planar orbits
- Generic orbits confined away from poles
- Orbits parametrised by angular momentum Φ
- Unstable under radial perturbations
- Some orbits have non-fixed azimuthal direction (Lense-Thirring effect)

Overview

The Myers-Perry spacetime

The 5D Myers-Perry black hole is the simplest generalisation of the Kerr spacetime. It describes a rotating spacetime in 5 dimensions with *two* planes of rotation.

$$ds^2 = -dt^2 + \frac{\mu}{\rho^2}\left(dt + a\sin^2\theta d\phi + b\cos^2\theta d\psi\right)^2 + \frac{\rho^2}{4\Delta}dx^2 + \rho^2 d\theta^2 + (x+a^2)\sin^2\theta d\phi^2 + (x+b^2)\cos^2\theta d\psi^2$$

In general, a Myers-Perry black hole describes a stationary black hole in d dimensions rotating in $N \equiv \lfloor \frac{d-1}{2} \rfloor$ independent planes.

Conserved quantities

The Myers-Perry metric is independent of the coordinates $\{t, \phi, \psi\}$, so the corresponding vector fields are isometries. The associated momenta are conserved quantities along geodesics.

In addition, there is an additional conserved quantity whose existence can be traced to the existence of a second order Killing tensor.

The conserved quantities are $\{E, \Phi, \Psi, K\}$.

Geodesic equations for null geodesics in 5D MP

The geodesic equations are a complicated, coupled set of ODEs:

$$\rho^2 \dot{t} = E\rho^2 + \frac{\mu(x+a^2)(x+b^2)}{\Delta}\mathcal{E}$$

$$\left(\rho^2 \dot{x}\right)^2 = 4\Delta \left[xE^2 + (a^2 - b^2)\left(\frac{\Phi^2}{x+a^2} - \frac{\Psi^2}{x+b^2}\right) - K\right] + 4\mu(x+a^2)(x+b^2)\mathcal{E}^2$$

$$\left(\rho^2 \dot{\theta}\right)^2 = E^2(a^2\cos^2\theta + b^2\sin^2\theta) + K - \frac{\Phi^2}{\sin^2\theta} - \frac{\Psi^2}{\cos^2\theta}$$

$$\rho^2 \dot{\phi} = \frac{\Phi}{\sin^2\theta} - \frac{\mu a(x+b^2)}{\Delta}\mathcal{E} - \frac{(a^2-b^2)\Phi}{x+a^2}$$

$$\rho^2 \dot{\psi} = \frac{\Psi}{\cos^2\theta} - \frac{\mu b(x+a^2)}{\Delta}\mathcal{E} + \frac{(a^2-b^2)\Psi}{x+b^2}$$

Constraints from the θ equation

Positivity of the geodesic equation for θ places constraints on the allowed values of $\{\theta, \psi\}$.

$$\left(\rho^2\dot{\theta}\right)^2 = E^2(a^2\cos^2\theta + b^2\sin^2\theta) + K - \frac{\Phi^2}{\sin^2\theta} - \frac{\Psi^2}{\cos^2\theta}$$

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Conclusion

Generic orbits are confined away from the poles *and* the equator!

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Conclusion

Generic orbits are confined away from the poles *and* the equator!

This also places constraints on the conserved quantities $\{K, \Phi, \Psi\}$.

Constant radius orbits

Insisting on constant radius orbits places additional constraints on the conserved quantities. Recall

$$\left(\rho^2\dot{x}\right)^2 = \chi = 4\Delta\left[xE^2 + (a^2 - b^2)\left(\frac{\Phi^2}{x+a^2} - \frac{\Psi^2}{x+b^2}\right) - K\right] + 4\mu(x+a^2)(x+b^2)\mathcal{E}^2$$

We can solve the equations $\chi = \frac{d\chi}{dx} = 0$ simultaneously for two of the given parameters, say x and K.

Procedure

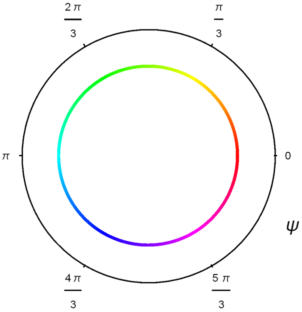
The solutions are messy. In practice, we follow this procedure:

1. Decide on a particular black hole (choose μ, a, b)
2. Select appropriate values of Φ, Ψ (consistent with the constraints from the θ equation)
3. Use Φ and Ψ to determine x and K
4. Numerically integrate and plot

Visualising the orbits

In order to visualise the orbits in 3-dimensions, we 'project out' the additional ψ coordinate.

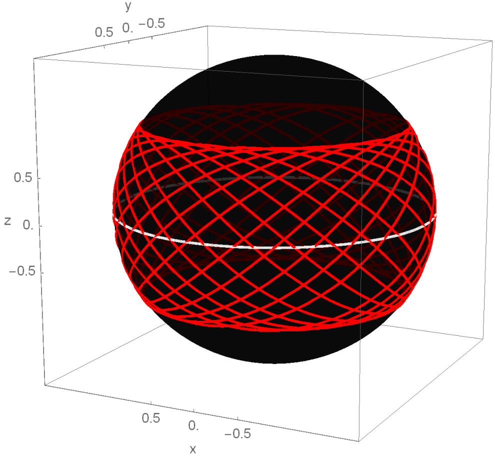
We don't lose any information, however, since we can encode the extra dimension with *colour*!



Overview

Kerr orbits

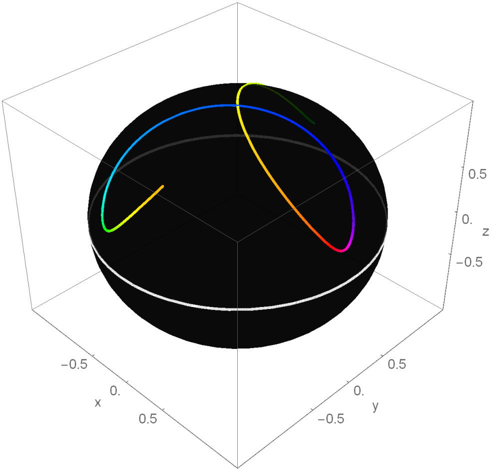
Many latitudinal oscillations of an orbit with $\Phi = 2$ and $\Psi = 0$.



$(a, b) = (0.5, 0)$.

"New" Kerr orbits

An orbit with $\Phi = 0$ and $\Psi = 0.5$:

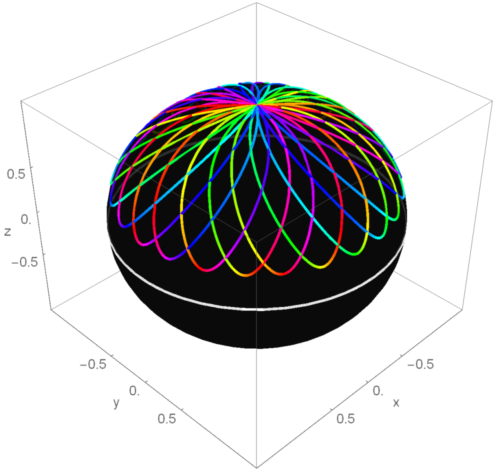


Note: orbit is confined away from equator.

$(a, b) = (0.8, 0)$.

“New” Kerr orbits

An orbit with $\Phi = 0$ and $\Psi = 0.5$:

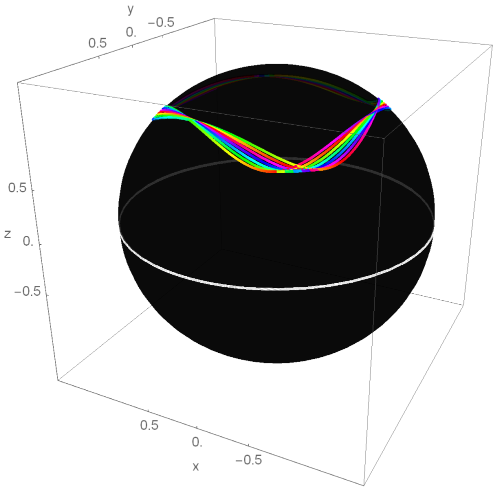


Note: orbit is confined away from equator.

$(a, b) = (0.8, 0)$.

Two planes of rotation

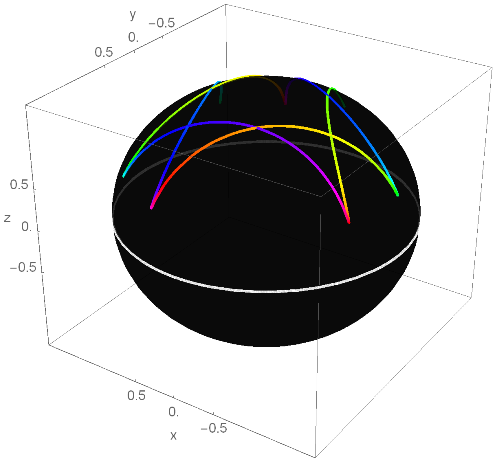
An orbit with $\Phi = 1$ and $\Psi = -1$.



This orbit is confined away from both the equator and the poles.

$(a, b) = (0.6, 0.3)$.

Cusps!



The appearance of cusps may seem counter-intuitive (geodesics should be differentiable!), but it is really just an artifact of our projection.

$(a, b) = (0.9, 0.1)$ and $\Phi = \Psi = 0.8$.

Features of photon orbits in Myers-Perry

- Non-planar orbits
- Generic orbits confined away from equator *and* poles
- Cusps
- Orbits parametrised by angular momenta Φ and Ψ
- Unstable under radial perturbations (suspected)
- Some orbits have non-fixed azimuthal and "other" direction (5D Lense-Thirring effect)

Overview

Further work

There are a variety of areas I want to explore

- Animation of orbits
- Arbitrary dimension Myers-Perry black holes
- Black ring?
- Black Saturn?

Thanks for listening!